Orbital Precession without GRT

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The anomalous precession of the planet Mercury's orbit puzzled scientist for decades. Einstein developed a curved space model of gravitation (General Relativity) and showed that the precession of the planets could be explained very accurately with this model. This fact combined with the prediction and subsequent observation of the gravitational bending of light rays, made GR one of, if not the most, highly accepted theories in science. In this paper I will show that the precession of the planets can be explained with nothing more then Special Relativity, Newtonian derived formulas and simple mathematical relationships. The result is extremely accurate.

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1. Introduction

The orbital precession of the planet Mercury has been known for decades. Prior to the advent of Special Relativity Theory (SRT), attempts to explain the precession were generally discarded for various reasons. [1] Attempts to explain the anomalous precession after the advent of SRT and prior to the development of General Relativity Theory (GRT) appear absent from the literature. Recently, a few papers have appeared attempting to explain the anomalous precession using some modifications/combinations of SRT and Newtonian gravity. Biswas [2] has shown that a Lorentz covariant modification of classic Newtonian gravitational potential can yield the correct precession. Others have made more radical assumptions such as modifying the gravitation field equations to parallel the field equations of Electromagnetism [3,4] or assuming a relativistic Lagrangian that looks surprisingly like a Schwarzschild metric [5]. The present consensus among 'mainstream' physicists seems to be that it simply cannot be explained with SRT [6].

The fact that the relativistic effects are very small in planetary motion would lead one to believe that some simple approximation involving Newtonian mechanics combined with SR should at least give a good approximation to orbital precession. It turns out that this can in fact be done with simple mathematics and a couple of well known and universally accepted physical formulas. No assumptions regarding the nature of the gravitational field or the geometry of space are required.

2. Background Physics

The energy of a relativistic particle in motion is given by:

$$E = m_0c^2 + \tfrac{1}{2}m_0v^2 + \tfrac{3}{8}m_0v^4 / 8c^2 + \tfrac{5}{16}m_0v^6 / c^4 \ldots \qquad (1)$$

In the discussion to follow, the kinetic energy of the particle will be:

$$T = \tfrac{1}{2}m_0v^2 + \tfrac{3}{8}m_0v^4 / c^2 \qquad (2)$$

Additional terms become relevant only at extreme velocities.

For the remainder of this discussion, unless specifically stated otherwise the symbol m will be equal to the rest mass m_0.

From Newtonian mechanics, the total velocity of an object moving under the influence of a central force is given by [7]:

$$v^2(r) = (L^2 / m^2P^2)\left[(\varepsilon^2 - 1) + 2P / r\right] \qquad (3)$$

where $P = L^2 / GMm^2$ and L is the total constant angular momentum and ε is the eccentricity of the orbit. In deriving Eq. (3), the v_θ component of the total velocity cancels out, and hence (3) can be considered a one dimensional, or scalar, equation. Eqs. (2) and (3) are all the physics that is needed. The rest is mathematics.

3. The Math

To develop the necessary mathematical relationships needed we first explore the relationship between the Lagrangian operators on general power functions. The Lagrangian operators are:

$$L_t = \frac{d}{dt}\frac{\partial}{\partial v} \quad \text{and} \quad L_x = \frac{\partial}{\partial x} \qquad (4)$$

In one dimensional mechanics, when given the initial conditions one can determine both the position x as a function of the time t: $x(t)$. And one can find the velocity v as a function of time: $v(t)$. Similarly time can be written (by taking the inverse function) as a function of position $t(x)$, and hence velocity can be written as a function of position $v(x)$.

Consider a general function of the form $a + kv^n$, where n is a positive integer ≥ 2 and a is a constant. The function can be written as:

$$f(t) = a + kv^n(t) = f(x) = a + kv^n(x) \text{ where } x = x(t)$$

Applying L_t to $f(t)$ we find:

$$\frac{d}{dt}\frac{\partial}{\partial v}(a + kv^n) = \frac{d}{dt}(nkv^{n-1}) = n(n-1)kv^{n-2}\frac{dv}{dt} \qquad (5)$$

Applying L_x to $f(x)$ we find:

$$\frac{\partial}{\partial x}\left[kv^n(x)\right] = nkv^{n-1}(x)\frac{\partial v}{\partial x} = nkv^{n-2}\frac{dx}{dt}\frac{\partial v}{\partial x} = nkv^{n-2}\frac{dv}{dt} \qquad (6)$$

In (6), kv^{n-1} was written as $kv^{n-2}\dfrac{dx}{dt}$ and $\dfrac{dx}{dt}\dfrac{\partial v}{\partial x}$ was replaced with $\dfrac{dv}{dt}$. From (5) and (6) it is clear that:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[kv^n(t)\right]=(n-1)\frac{\partial}{\partial x}\left[kv^n(x)\right] \tag{7}$$

Equation (7) is not an equation of motion or a 'law of physics'; it is a simple mathematical equality valid for any functions where the inverse of $x(t)$ exists and the functions $v(t)$ and $v(x)$ are differentiable. In any real world physics problem both $kv^n(t)$ and $kv^n(x)$ will be kinetic energy functions and:

$$kv^n(t)=kv^n(x) \quad \text{where} \quad x=x(t) \quad . \tag{8}$$

We can derive the classical Lagrangian equation of motion from (7) by assuming:

a. The total energy of a body moving is the sum of its kinetic energy T, and its potential energy V

b. The total energy is a constant

From (7) with $n=2$:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[T(t)\right]=\frac{\partial}{\partial x}\left[T(x)\right] \tag{9}$$

Eq. (9) is valid regardless of any constants added to T, so we can write:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[T(t)\right]=\frac{\partial}{\partial x}\left[-E_{\mathrm{T}}+T(x)\right] \quad .$$

where E_{T} is the total energy, which is assumed constant.

$-E_{\mathrm{T}}+T(x)$ is just $-V(x)$, the potential energy which is assumed to be a function only of x. Since L_t operating on $V(x)$ is zero and L_x operating on $T(t)$ is zero we can write (7) as:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[T(t)-V(x)\right]=\frac{\partial}{\partial x}\left[T(t)-V(x)\right] \tag{10}$$

Eq. (10) is in the form of the classical Lagrangian "equation of motion".

When the kinetic energy can be written as a function of $v(t)$ only, and the potential energy is a function of x only, and the total energy is constant, we conclude that:

$$\begin{aligned}\frac{d}{dt}\frac{\partial}{\partial v}\left[T(t)\right]&=(n-1)\frac{\partial}{\partial x}\left[T(x)\right]\\&=(n-1)\frac{\partial}{\partial x}\left[-E_{\mathrm{T}}+T(x)\right]=(n-1)\frac{\partial}{\partial x}\left[-V(x)\right]\end{aligned} \tag{11}$$

We can use (9) and (3) to generate the gravitational force on an orbiting body. Using:

$$T(t)=\tfrac{1}{2}mv^2(t) \quad \text{and} \quad T(r)=\tfrac{1}{2}mv^2(r)$$

and substituting (3) for $v^2(r)$ then differentiating we get:

$$ma=-GMm/r^2 \tag{12}$$

This is the classic Newtonian formula.

We just showed that, knowing the velocity as a function of r (7) allows one to generate the potential energy function. In most real problems $v(x)$ is not known, rather some potential field or potential energy function is known. Although the potential energy must be equal to $-\left[-E_{\mathrm{T}}+T(x)\right]$, the functional form may not resemble $v^n(t)$ and Eq. (6) could be written as:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[kv^n(t)\right]=(n-1)\frac{\partial}{\partial x}k_1\phi_n(x) \tag{13}$$

k_1 in (9) could represent any constant and in the case of gravitational potential it is the mass or reduced mass of the orbiting body.

$k_1\phi_n(x)$ is a spatially distributed energy function and the force it exerts on any classical Newtonian particle (a particle whose kinetic energy is $\frac{1}{2}mv^2$) would be determined by (13) with $n=2$ regardless of the functional form of $\phi_n(x)$.

In particular if, $k_1\phi_n(x)=E_{\mathrm{T}}-V(x)=T(x)=\frac{3}{8}m_0v^4/c^2$, a classical Newtonian particle would behave according to:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right)=\frac{\partial}{\partial x}\left[\frac{3}{8}mv^4(x)/c^2\right] \tag{14}$$

In simple Newtonian terms if some strange particle whose kinetic energy were equal to $3m_0v^4(t)/8c^2$ wandered into a potential energy field of $E_{\mathrm{T}}-V(x)=\frac{3}{8}m_0v^4(x)/c^2$, Newton's law of motion would have to be modified to: $mA=3F$, where $A=\dfrac{1}{m}\dfrac{d}{dt}\dfrac{\partial}{\partial v}\left(\frac{3}{8}mv^4(t)/c^2\right)$.

Any real particle whose kinetic energy is given by $mv^2/2$ would of course obey Newton's law ($ma=F$), regardless of the functional form of the potential energy function.

The above reasoning applies to a one-dimensional system, and by inference it could be applied to any multidimensional Cartesian coordinate system. In polar coordinates, the square of the velocity is not a function of a single variable, and conventional Lagrangian methods must be used; however, because (3) is a function only of r, the magnitude of $\mathbf{r}$, the above reasoning can be used to determine an effective potential field.

We now have the tools necessary to solve the problem of orbital precession. The gravitational field depends only on r, so we can use the same one-dimensional reasoning. Using (2) and (7) we get the equalities:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[\frac{1}{2}mv^2(t)\right]=\frac{\partial}{\partial r}\left[\frac{1}{2}mv^2(r)\right]$$

and $$\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{3}{8}mv^4(t)/c^2\right]=3\frac{\partial}{\partial r}\left[\tfrac{3}{8}mv^4(r)/c^2\right]$$

Combining terms we get the equality:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{1}{2}mv^2(t)\right]+\frac{\partial}{\partial t}\frac{\partial}{\partial v}\left[\tfrac{3}{8}mv^4(t)/c^2\right]$$

$$=\frac{\partial}{\partial r}\left[\tfrac{1}{2}mv^2(r)\right]+3\frac{\partial}{\partial r}\left[\tfrac{3}{8}mv^4(r)/c^2\right]$$

We don't know $v(r)$ exactly, but for most problems, such as the orbit of the Planet Mercury, we know that it very closely approximates the classical $v(r)$. We will then use Eq. (3) on the right side. Squaring (3) to get $v^4(r)$ and differentiating we get.

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{1}{2}mv^2(t)\right]+\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{3}{8}mv^4(t)/c^2\right]= -\frac{GMm}{r^2}-\frac{3GMmk}{2r^2}-9\frac{G^2M^2m}{r^3c^2} \tag{15}$$

where $$k=3G^2M^2m^2(\varepsilon^2-1)\Big/L^2c^2 \tag{16}$$

This k is very small compared with the first term, and has no effect on the orbital precession, but may have some interesting consequences that we shall discuss later.

To simplify (15) into a solvable physic problem we note that it can be considered to represent a simple Newtonian particle, with an 'extra' acceleration term $\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{3}{8}mv^4(t)/c^2\right]$, being acted upon by a force equal to: $-\frac{GMm}{r^2}-\frac{3GMmk}{2r^2}-9\frac{G^2M^2m}{r^3c^2}$.

The $v(t)$ function in both terms on the left of (15) must be the same function, and we can seek a solution for the first term. The second term is very small compared to the first, and we can convert it to a force by again using (3) as an approximation. From (11) we determine the equivalent force to be:

$$(n-1)\frac{\partial}{\partial r}\left[\tfrac{3}{8}mv^4(r)/c^2\right]=(n-1)\left[-\frac{GMmk}{2r^2}-3\frac{G^2M^2m}{r^3c^2}\right] \tag{17}$$

To determine n in Eq. (17), we note that after this substitution, the only acceleration term left is that of the simple Newtonian particle: $\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{1}{2}mv^2(t)\right]$.

This force will be acting on this simple Newtonian particle and therefore, from the previous reasoning, n must be 2. We can then make the substitution, rearrange terms and arrive at a solvable equation:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left[\tfrac{1}{2}mv^2(t)\right]=-\frac{GMm}{r^2}-\frac{GMmk}{r^2}-6\frac{G^2M^2m}{r^3c^2}$$

We could either ignore the second term, or combine it with the first term, effectively skewing the Gravitational constant. Interestingly, it is a function of eccentricity, and depending on the orbit, it can be either positive, negative, or zero. We can proceed to solve (18), ignoring the second term both because it is very small and because it has no effect on the precession. Using conventional Lagrangian methods in polar coordinates, v^2 gets replaced by $(dr/dt)^2+(rd\theta/dt)^2$, resulting in two equations, one from the r coordinate and one from the θ coordinate

Since we want to determine orbital precession, we would like to solve this equation for $r(\theta)$. Fortunately, the method for solving it can be found in many textbooks and many online sites [7]. In summary, the solution of the θ equation is independent of the nature of the potential field and results in conservation of angular momentum designated as L. The following conversions are then used on the radial equation:

$$\frac{d}{dt}=\frac{d\theta}{dt}\frac{d}{d\theta}=\frac{L}{mr^2}\frac{d}{d\theta}$$

and $$\frac{1}{r^2}\frac{dr}{d\theta}=-\frac{d(1/r)}{d\theta}$$

and $$u\equiv 1/r$$

Applying these conversions yields:

$$u^2\left(d^2u/d\theta^2+u\right)=u^2/l+6(h/cl)^2u^3 \tag{19}$$

where $h=L/m$, the angular momentum per unit mass and $l=h^2/GM$. The c is of course the speed of light. The l is a measurable property of an orbit, and is designated the 'semi-latus rectum'. It is related to the eccentricity.

Simplifying Eq (19) yields:

$$d^2u/d\theta^2+\left[1-6(h/cl)^2\right]u=1/l \tag{20}$$

Eq (20) can be solved exactly by letting $u=A+B\cos(\Delta\theta)$. Doing the math gives $\Delta=\sqrt{1-6(h/cl)^2}$, from which we can directly determine the orbital precession.

$$(h/cl)^2=GM/c^2l$$

For the planet Mercury, $l=55.443\times10^6$ km, $(h/cl)^2=GM/c^2l$ and $GM/c^2=1.475$ km, giving $\Delta=$ 0.999999920188298314. The precession per revolution in radians is: $2\pi/\Delta-2\pi=5.014717513978\times10^{-7}$ radians. Converting to arc seconds, the precession becomes: 0.1034359736404089.

Mercury orbits the Sun 414.9378 times in one Earth century, so the precession per century is: 42.9195 arc seconds per Earth century, in excellent agreement with observation and GRT

Conclusions

The precession observed in the orbit of Mercury can be explained with nothing more than well-accepted physical formulas, mathematical identities, and relatively simple mathematics. No assumptions about the nature of the gravitational field are required and space is treated as 'flat'. Although the above discussion is based on approximations, the small relativistic components should and do make it very accurate. One could infer from the above discussion that gravity acts on an objects total energy and not just rest mass. The additional energy is then treated as additional kinetic energy. No alterations of Newtonian gravity are required.

In this paper the Lagrangian operators were used to develop mathematical identities. It is clear that a modification of the Lagrangian method was required based on the functional form of energy equations. One can only wonder what other problems in physics may benefit from a reevaluation of the application of the Lagrangian method to non-Newtonian energy functions.

In the calculations there appears a curious term that deserves some exploration. The k term is orbital-path related. It skews the gravitational constant and this skewing can be positive or negative or zero depending on the eccentricity of the orbit. For the Pioneer space probe $\varepsilon = 1.7372$ and k is positive resulting in an increase in the gravitational force. Prior to being subjected to the 'sling shot effect', its eccentricity was most likely much smaller. This would have resulted in an effective increase in G after it reached its escape velocity, and may be at least partially responsible for the pioneer anomaly.

By including the k term in the calculation, one would expect slight variations from predicted orbits. NASA often uses the sling-shot effect to accelerate space craft. It is common to flyby a planet in a particular orbital path in order to change the orbit relative to the Sun (often substantially increasing velocity relative to the solar system Barycenter) and substantially changing the eccentricity of the crafts solar orbit. NASA has in fact measured small anomalies in the majority of Earth Flybys [8]. Based on the formula developed here, one would expect small anomalies in the resulting solar orbit. Although a substantial amount of orbital data relative to the Earth has been published, there appears to be no data published relative to the change in the solar orbit. Such data would be necessary in order to determine if the method used here could shed some light on the anomalies.

This author is preparing further papers to show that both the bending of light and the gravitational red shift can be explained using reasoning similar to that described here.

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